

Falling body, the example in below show you how to calculate on a free falling sphere with respect to the gravitation and the resistance from the air. The sphere is made of iron, with a diameter of 10 cm.

acc := 9.81	The earth acceleration (g=9.81)	Describing differetialequation Energy balance
m := 4.2	The mass of the sphere (iron)	
A := 0.007854	The frontarea of the sphere	$\frac{m \cdot s^2}{2} = m \cdot acc \cdot s - \frac{\rho \cdot A \cdot cw \cdot s^2 \cdot s}{2}$
$\rho := 1.2$	The density of the air	
cw := 0.2	The cw value of the sphere, a sort of shape factor	

Given

$$\frac{d}{dt}s(t) = \sqrt{\frac{2 \cdot acc \cdot m \cdot s(t)}{\rho \cdot A \cdot cw \cdot s(t) + m}}$$

The diff equation, here a combine the law that $F \cdot s$ is the energy, the law of the dynamic pressure and the law of kinetic energy. Then I set up a energy balance. And I got the describing unlinjeur differential equation of the first order.

$$s(0) = 0.001$$

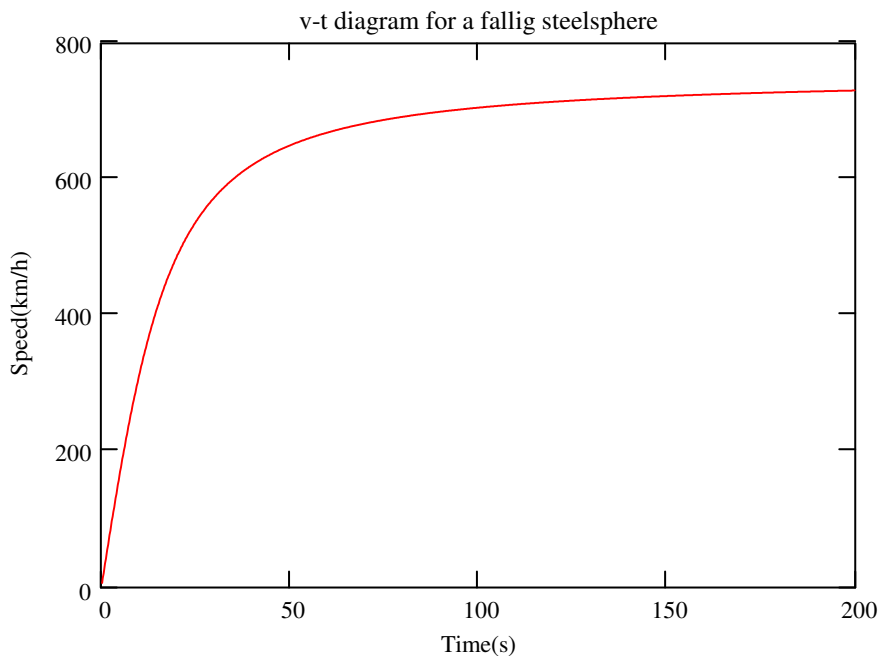
I set a small error in the initalvalue, so the calculating process can begin.

$$s := \text{Odesolve}(t, 200, 1000)$$

Solving the diff up to 200 s, and in 1000 steps

$$v(t) := \frac{d}{dt}s(t) \cdot 3.6$$

Calculate the speed and multiplying it with 3.6 to get the speed in km/h



Now the next question are how much the steel sphere is rise in temperature when the sphere reaches a stabile velocity. Here a use the law that $F \cdot v$, force * speed is the effect. And then I can set it equal to the convection effect.

$P = \alpha \cdot A_{\text{matel}} \cdot (T_{\text{sphere}} - T_{\text{air}})$. I know of the equation abowe that the top speed is about 209 m/s. The dynamic pressure is given of $\rho \cdot v^2 / 2$. If a multiplying the dynamic pressure with cw value and than multiplying it with the area of the sphere and multiplying it to the speed. I got the effect who I put into the steelsphere. Then I set this equation equal to the convection effect, when the temp of the air I supposed to be 20 degrees. The convection area is calculated by the matelarea of the sphere. The convection constant alpha gives of: $7.15 \cdot \text{falling speed}^{0.78}$. Then you can solve the temperature of the falling sphere.

$$(1) P = F \cdot v$$

$$(2) P = \alpha \cdot A_{\text{matel}} \cdot (T_{\text{sphere}} - T_{\text{air}})$$

$$A_m := 0.05^2 \cdot \pi \cdot 4$$

$$v := 209$$

$$\alpha := 7.15 \cdot v^{0.78}$$

$$T_{\text{air}} := 20$$

$$P1 := \frac{\rho \cdot A \cdot c_w \cdot v^2}{2} \cdot v$$

$$T_{\text{sphere}} := \frac{P1}{\alpha \cdot A_m} + T_{\text{air}}$$

$$T_{\text{sphere}} = 613.664$$

The calculate temperature is about 614 degrees on the steel sphere when the sphere reaches a stabile velocity. This type to calculate the temperature, I believe it can use when you want to calculate on the space shuttle penetrates the atmosphere, in entrance to the planet earth. A way to determine how much convection effect I put into the space shuttle is to messure the reaccelaration and you know the velocity of the shuttle and the mass.

$P = m \cdot a \cdot v$, set this equal to the convection effect eq(2).

$$\frac{d^2}{dt^2} s(t) = \text{acc} - \frac{\left(\frac{d}{dt} s(t) \right)^2 \cdot A \cdot \rho \cdot c_w}{2m}$$

This is I suppose the more correct diff equation to describe a falling body. The diff is of a second order unlinjeir diff eqv. You can put this equation instead of that abowe. But I want to show that there is many way you can attac a problem. With this equation the max speed on the sphere is about 209 m/s. Same as the end velocity in the diff in the further side.